Criminal Network Formation and Optimal Detection Policy: The Role of Cascade of Detection

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Criminal networks are widely observed

- Mafia
- Terrorist networks
- Corruption networks
Research Question

- What is the optimal detection policy in the presence of endogenous network formation among criminals?
- How does the cascade of detection affect criminal network formation and social welfare?
We consider two dimensions of detection policy
- Allocation of detection resource
- Degree of cascade

Higher degree of cascade of detection may backfire

Optimal budget allocation is highly asymmetric among ex ante identical agents
Our timing structure follows Baccara and Bar-Issac (2008)

- Players form link with each other
- Payoff is realized
- The government announces allocation of detection resources and degree of cascade
- Given the criminal network, players choose the level of criminal activity
Model

1. Set of players: \( N = \{1, 2, \ldots, n\} \)
2. Probability of player \( i \) being directly detected: \( \beta_i \in [0, 1] \)
3. The government allocates a fixed detection budget \( B \in \mathbb{R}_+ \)
   \[
   \sum_{i=1}^{n} \beta_i \leq B
   \]
4. Players are ranked such that
   \[
   \beta_1 \leq \beta_2 \leq \ldots \leq \beta_n
   \]
5. \( \beta \equiv (\beta_1, \beta_2, \ldots, \beta_n) \)
Link formation

- $G$: set of $n$-by-$n$ $(0, 1)$-matrices with zeros on the diagonal
- $G_i$: set of $n$-by-$1$ $(0, 1)$-vectors with $i$-th element to be zero
- $g_i \in G_i$: linking decision by player $i$
- $g \in G$: A collection of linking choices by all players
- $\overline{G}$: set of $n$-by-$n$ symmetric $(0, 1)$-matrices with zeros on the diagonal

Link formation requires bilateral agreement. For any $g \in G$, $g$ induces a criminal network $\overline{g}(g) \in \overline{G}$ such that

$$\overline{g}(g) = \min(g, g')$$

- No explicit linking cost
Degree of Cascade

- Distance $d_{ij}$ between player $i$ and $j$ is the length of the shortest path connecting $i$ and $j$
- Probability of player $i$ not being detected

$$p_i(g; \beta, d) = \prod_{j \in N, d_{ij} \leq d} (1 - \beta_j)$$

with
- $d = 0 \rightarrow$ no cascade of detection
- $d = 1 \rightarrow$ limited cascade of detection
- $d = n \rightarrow$ full cascade of detection
Degree of Cascade: Example

- $p_1(\bar{g}; \beta, 0) = 1 - \beta_1$
- $p_1(\bar{g}; \beta, 1) = \prod_{i=1}^{4}(1 - \beta_i)$
- $p_1(\bar{g}; \beta, 6) = \prod_{i=1}^{6}(1 - \beta_i)$
Given $\bar{g} \in \bar{G}$, player $i$ chooses effort level $x_i \in \mathbb{R}_+$

Player $i$’s payoff

$$\pi_i(x, \bar{g}; \beta, \lambda, d) = p_i(\bar{g}; \beta, d) \cdot \left( x_i - \frac{1}{2}x_i^2 + \lambda \sum_{j=1}^{n} \bar{g}_{ij}x_ix_j \right)$$

where $\lambda \in (0, \frac{1}{n-1})$ and $x \equiv (x_1, x_2, \ldots, x_n)$
Strategy and Strategy Profile

- $X$: set of all mappings from $\overline{G}$ to $\mathbb{R}_+$
- Player $i$'s strategy is a pair of a linking choice $g_i \in G_i$ and an effort mapping $x_i(\cdot) \in X$
- Given a strategy profile $(x(\cdot), g)$, player $i$'s payoff

$$\Pi_i(x(\cdot), g) \equiv \pi_i(x(\overline{g}(g)), \overline{g}(g))$$
Definition

A **Nash equilibrium** is a strategy profile \((x^\ast(\cdot), g^\ast)\) such that

\[
\Pi_i(x^\ast(\cdot), g^\ast) \geq \Pi_i(x_i(\cdot), x_{-i}^\ast(\cdot), g_i, g_{-i}^\ast), \quad \forall i \in N, \ x_i(\cdot) \in X, \ g_i \in G_i.
\]

Definition

A **subgame-perfect Nash equilibrium** is a strategy profile \((x^\ast(\cdot), g^\ast)\) such that a Nash equilibrium is played for every subgame.
Equilibrium Definition II

Definition (Hiller, 2014)

A pairwise stable Nash equilibrium is a strategy profile \((x^*(\cdot), g^*)\) such that

1. \((x^*(\cdot), g^*)\) is a subgame-perfect Nash equilibrium.
2. There is no profitable bilateral deviation at the stage of link formation. For any \((i, j)\)-pair such that \(g(g^*)_{ij} = 0\) (\(i \neq j\)),

\[
\Pi_i(x^*(\cdot), g^* \oplus (i, j) \oplus (j, i)) > \Pi_i(x^*(\cdot), g^*)
\]

implies

\[
\Pi_j(x^*(\cdot), g^* \oplus (i, j) \oplus (j, i)) < \Pi_j(x^*(\cdot), g^*).
\]
A network $\overline{g}' \in \overline{G}$ is **obtainable** from $\overline{g} \in \overline{G}$ via deviations by a nonempty $S \subset N$ if

1. $\overline{g}_{ij} = 0$ and $\overline{g}'_{ij} = 1$ implies $i, j \in S$;
2. $\overline{g}_{ij} = 1$ and $\overline{g}'_{ij} = 0$ implies $\{i, j\} \cap S \neq \emptyset$. 
Obtainability: Example
A subgame-perfect Nash equilibrium \((x^*(\cdot), g^*)\) is **strongly stable** if for any nonempty \(S \subseteq N\), \(\overline{h} \in \overline{G}\) that is obtainable from \(\overline{g}(g^*)\) via deviations by \(S\), and \(i \in S\) such that

\[\pi_i(x^*(\overline{h}), \overline{h}) > \pi_i(x^*(\overline{g}(g^*)), \overline{g}(g^*)),\]

there exists \(j \in S\) such that

\[\pi_j(x^*(\overline{h}), \overline{h}) < \Pi_j(x^*(\overline{g}(g^*)), \overline{g}(g^*)).\]
Lemma (Ballester, et al., 2006)

Given a criminal network $\overline{g} \in \overline{G}$, if $\lambda \in (0, 1/(n - 1))$, there exists a unique interior Nash equilibrium for the stage game at the second period. In particular,

$$x(\overline{g}) = (I - \lambda \overline{g})^{-1} \cdot 1,$$

where $I$ is an $n$-dimensional identity matrix and $1$ is a 1-by-$n$ vector with all elements equal to one. Moreover, player $i$’s equilibrium payoff is given by $p_i(\overline{g}) x_i^2(\overline{g})/2$. 
Equilibrium Characterization: No Cascade

Proposition

*If there is no cascade of detection \((d = 0)\), there exists a generically unique pairwise stable Nash equilibrium in which agents form a complete network.*
Equilibrium Characterization: Full Cascade

**Lemma**

Under full cascade of detection \(d = n\), each component of the criminal network is complete in a pairwise stable Nash equilibrium.

**Lemma**

In any strongly stable Nash equilibrium, the equilibrium partition of agents “preserves” the order of detection probability

\[
\left\{ \{1, ..., n_1\}, \{n_1 + 1, ..., n_1 + n_2\}, ..., \{\sum_{i=1}^{k-1} n_i + 1, ..., \sum_{i=1}^k n_i\} \right\}
\]

where \(n \equiv \sum_{i=1}^k n_i\) and agents are labeled such that \(\beta_1 \leq ... \leq \beta_n\).
Proposition

There exists a generically unique strongly stable Nash equilibrium with the equilibrium partition \( \{ \{1, 2, \ldots, n_0\}, \{n_0 + 1\}, \{n_0 + 2\}, \ldots, \{n\} \} \) and

\[
n_0 = \max \left\{ \arg \max_{k \in \mathbb{N}} \pi^k \right\},
\]

where \( \pi^k \) is the individual payoff of a complete component formed by the first \( k \) agents,

\[
\pi^k = \frac{1}{2} \left( \frac{1}{1 - (k - 1)\lambda} \right)^2 \prod_{i=1}^{k} (1 - \beta_i).
\]
Equilibrium Characterization: Full Cascade

A numerical example: \( n = 10; \beta_k = k/20; \lambda = 0.08 \)
Equilibrium Characterization: Partial Cascade

Proposition

Those players who are isolated in the strongly stable Nash equilibrium under full cascade of detection remain isolated in any pairwise stable Nash equilibrium under partial cascade of detection.
The government’s decision problem

\[
\min_{\beta \in \mathbb{R}^n_+ : \sum_i^n \beta_i \leq B} \sum_{i=1}^n \left( x_i(\beta, \lambda) - \frac{1}{2} x_i^2(\beta, \lambda) + \lambda \sum_{j=1}^n g_{ij}(\beta, \lambda) x_i(\beta, \lambda) x_j(\beta, \lambda) \right)
\]

Equivalently,

\[
\min_{\beta \in \mathbb{R}^n_+ : \sum_i^n \beta_i \leq B} n_0(\beta)
\]
Proposition

Under full cascade of detection, the government can keep each agent isolated in the strongly stable Nash equilibrium if and only if

\[ B > B_1 \equiv n - 1 - \sum_{k=2}^{n} \left( \frac{1 - (k - 1)\lambda}{1 - (k - 2)\lambda} \right)^2, \]

and the optimal allocation of the detection budget is given by \( \beta_1 = 0 \) and

\[ \beta_k = 1 - \left( \frac{1 - (k - 1)\lambda}{1 - (k - 2)\lambda} \right)^2 + \frac{B - B_1}{n - 1}, \quad k = 2, 3, \ldots, n. \]
Corollary

Under full cascade of detection, the government can keep the size of the largest component of the criminal network in the strongly stable Nash equilibrium to be $S \in \{2, 3, \ldots, n-1\}$ if and only if

$$B > B_S \equiv n - S - \sum_{k=S+1}^{n} \left( \frac{1 - (k - 1)\lambda}{1 - (k - 2)\lambda} \right)^2,$$

and the optimal allocation of the detection budget is given by $\beta_k = 0$ for $k \leq S$ and

$$\beta_k = 1 - \left( \frac{1 - (k - 1)\lambda}{1 - (k - 2)\lambda} \right)^2 + \frac{B - B_S}{n - S}, \quad k = S + 1, S + 2, \ldots, n.$$
Relation to the Literature

- Crime organizations and detection policy
  - Ballester, Calvo-Armengol, and Zenou (2006): “key-player” policy
  - Garoupa (2007): severe law enforcement could backfire
  - Baccara and Bar-Isaac (2008): optimal information structure

- Network games with local complementarities
  - Baetz (2014): one-sided link formation
  - Hiller (2014): two-sided link formation
  - Belhaj, Bervoets, and Deroian (2014): efficient network structures
Proposition

Those players who are isolated in the strongly stable Nash equilibrium under full cascade of detection \((d = n)\) remain isolated in any strongly stable Nash equilibrium under positive degree of cascade \((d \in \{1, 2, \ldots, n\})\).
Proposition

Let $B_n = 0$. If the detection budget $B \in [B_{\ell+1}, B_{\ell})$ for $\ell \in \{1, 2, \ldots, n-1\}$, the government can incentivize all agents to opt out if and only if $\frac{1-(B-B_{\ell+1})}{2(1-\ell\lambda)^2} < \pi_0$ with the allocation of the detection budget given by $\beta_k = 0$ for $k \leq \ell$, $\beta_{\ell+1} = B - B_{\ell+1}$, and

\[
\beta_k = 1 - \left( \frac{1-(k-1)\lambda}{1-(k-2)\lambda} \right)^2 \quad \text{for } k > \ell + 1.
\]

\[\text{\textsuperscript{a}}\text{Recall that } B_S \equiv n - S - \sum_{k=S+1}^{n} \left( \frac{1-(k-1)\lambda}{1-(k-2)\lambda} \right)^2 \text{ for } S \in \{1, 2, \ldots, n-1\}.\]
Conclusion

- Higher degree of cascade of detection may backfire. Higher degree of cascade of detection could yield a denser criminal network.
- Optimal budget allocation is highly asymmetric among ex ante identical agents.
- The results continue to hold under general degree of cascade of detection and introduction of outside option.
Thank you!