Criminal Network Formation and Optimal Detection Policy: The Role of Cascade of Detection

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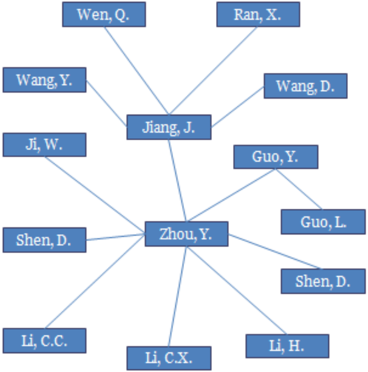
Motivation



Criminal networks are widely observed

- Mafia
- Terrorist networks
- Corruption networks

Research Question

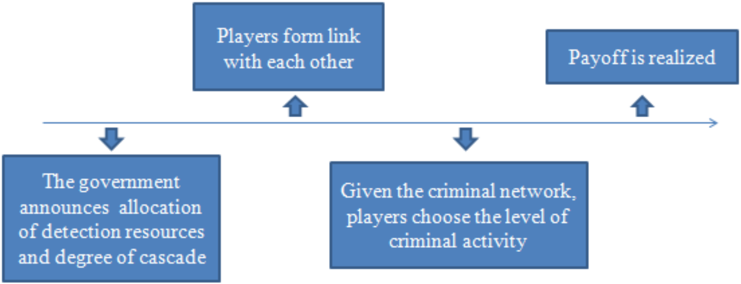
- What is the optimal detection policy in the presence of endogenous network formation among criminals?
- How does the cascade of detection affect criminal network formation and social welfare?

Preview of the Results

- We consider two dimensions of detection policy
  - Allocation of detection resource
  - Degree of cascade
- Higher degree of cascade of detection may backfire
- Optimal budget allocation is highly asymmetric among ex ante identical agents

Time Line

Our timing structure follows Baccara and Bar-Issac (2008)



Model

1. Set of players: $N = \{1, 2, ..., n\}$
2. Probability of player i being directly detected: $\beta_i \in [0, 1]$
3. The government allocates a fixed detection budget $B \in \mathbb{R}_+$

$$\sum_{i=1}^{n} \beta_i \leq B$$

4. Players are ranked such that

$$\beta_1 \leq \beta_2 \leq ... \leq \beta_n$$

5. $\beta \equiv (\beta_1, \beta_2, ..., \beta_n)$

Link formation

- G: set of n-by-n $(0, 1)$-matrices with zeros on the diagonal
- G_i: set of n-by-1 $(0, 1)$-vectors with i-th element to be zero
- $g_i \in G_i$: linking decision by player i
- $g \in G$: A collection of linking choices by all players
- $\overline{G}$: set of n-by-n symmetric $(0, 1)$-matrices with zeros on the diagonal
- Link formation requires bilateral agreement. For any $g \in G$, g induces a criminal network $\overline{g}(g) \in \overline{G}$ such that

$$\overline{g}(g) = \min(g, g')$$

- No explicit linking cost

Degree of Cascade

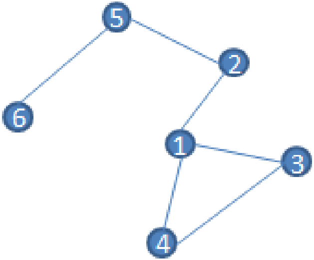
- Distance d_{ij} between player i and j is the length of the shortest path connecting i and j
- Probability of player i *not* being detected

$$p_i(\overline{g}; \beta, d) = \Pi_{j \in N, d_{ij} \leq d}(1 - \beta_j)$$

with

  - $d = 0 \rightarrow$ no cascade of detection
  - $d = 1 \rightarrow$ limited cascade of detection
  - $d = n \rightarrow$ full cascade of detection

Degree of Cascade: Example



- $p_1(\overline{g}; \beta, 0) = 1 - \beta_1$
- $p_1(\overline{g}; \beta, 1) = \Pi_{i=1}^{4}(1 - \beta_i)$
- $p_1(\overline{g}; \beta, 6) = \Pi_{i=1}^{6}(1 - \beta_i)$

Strategic Complementarity

- Given $\overline{g} \in \overline{G}$, player i chooses effort level $x_i \in \mathbb{R}_+$
- Player i's payoff

$$\pi_i(x, \overline{g}; \beta, \lambda, d) = p_i(\overline{g}; \beta, d) \cdot \left(x_i - \frac{1}{2} x_i^2 + \lambda \sum_{j=1}^{n} \overline{g}_{ij} x_i x_j \right)$$

where $\lambda \in (0, \frac{1}{n-1})$ and $x \equiv (x_1, x_2, ..., x_n)$

Strategy and Strategy Profile

- X: set of all mappings from $\overline{G}$ to $\mathbb{R}_+$
- Player i's strategy is a pair of a linking choice $g_i \in G_i$ and an effort mapping $x_i(\cdot) \in X$
- Given a strategy profile $(x(\cdot), g)$, player i's payoff

$$\Pi_i(x(\cdot), g) \equiv \pi_i(x(\overline{g}(g)), \overline{g}(g))$$

Equilibrium Definition I

Definition

A ***Nash equilibrium*** *is a strategy profile* $(x^*(\cdot), g^*)$ *such that*

$$\Pi_i(x^*(\cdot), g^*) \geq \Pi_i(x_i(\cdot), x^*_{-i}(\cdot), g_i, g^*_{-i}), \quad \forall i \in N, \; x_i(\cdot) \in X, \; g_i \in G_i.$$

Definition

A ***subgame-perfect Nash equilibrium*** *is a strategy profile* $(x^*(\cdot), g^*)$ *such that a Nash equilibrium is played for every subgame.*

Equilibrium Definition II

Definition (Hiller, 2014)

A ***pairwise stable Nash equilibrium*** *is a strategy profile* $(x^*(\cdot), g^*)$ *such that*

1. $(x^*(\cdot), g^*)$ *is a subgame-perfect Nash equilibrium*
2. *There is no profitable bilateral deviation* ***at the stage of link formation****. For any* (i, j)*-pair such that* $\overline{g}(g^*)_{ij} = 0$ $(i \neq j)$*,*

$$\Pi_i(x^*(\cdot), g^* \oplus (i,j) \oplus (j,i)) > \Pi_i(x^*(\cdot), g^*)$$

implies

$$\Pi_j(x^*(\cdot), g^* \oplus (i,j) \oplus (j,i)) < \Pi_j(x^*(\cdot), g^*).$$

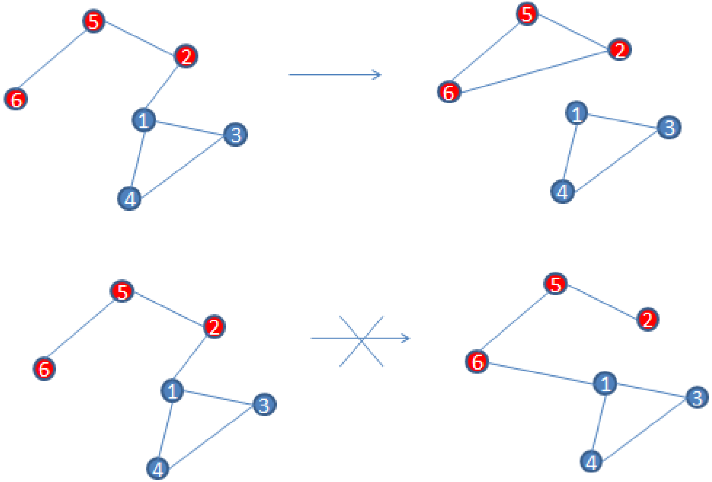
Obtainability

Definition (Jackson and van den Nouweland, 2005)

*A network $\overline{g}' \in \overline{G}$ is **obtainable** from $\overline{g} \in \overline{G}$ via deviations by a nonempty $S \subset N$ if*

1. $\overline{g}_{ij} = 0$ *and* $\overline{g}'_{ij} = 1$ *implies* $i, j \in S$;
2. $\overline{g}_{ij} = 1$ *and* $\overline{g}'_{ij} = 0$ *implies* $\{i, j\} \cap S \neq \emptyset$.

Obtainability: Example



Equilibrium Definition III

Definition (Jackson and van den Nouweland, 2005)

A subgame-perfect Nash equilibrium $(x^*(\cdot), g^*)$ *is* ***strongly stable*** *if for any nonempty* $S \subset N$, $\overline{h} \in \overline{G}$ *that is obtainable from* $\overline{g}(g^*)$ *via deviations by* S, *and* $i \in S$ *such that*

$$\pi_i(x^*(\overline{h}), \overline{h}) > \pi_i(x^*(\overline{g}(g^*)), \overline{g}(g^*)),$$

there exists $j \in S$ *such that*

$$\pi_j(x^*(\overline{h}), \overline{h}) < \Pi_j(x^*(\overline{g}(g^*)), \overline{g}(g^*)).$$

Equilibrium Characterization

Lemma (Ballester, et al., 2006)

Given a criminal network $\overline{g} \in \overline{G}$*, if* $\lambda \in (0, 1/(n-1))$*, there exists a unique interior Nash equilibrium for the stage game at the second period. In particular,*

$$x(\overline{g}) = (\mathbf{I} - \lambda \overline{g})^{-1} \cdot \mathbf{1},$$

where $\mathbf{I}$ *is an n-dimensional identity matrix and* $\mathbf{1}$ *is a 1-by-n vector with all elements equal to one. Moreover, player i's equilibrium payoff is given by* $p_i(\overline{g})x_i^2(\overline{g})/2$*.*

Equilibrium Characterization: No Cascade

Proposition

If there is no cascade of detection ($d = 0$), there exists a generically unique pairwise stable Nash equilibrium in which agents form a complete network.

Equilibrium Characterization: Full Cascade

Lemma

Under full cascade of detection ($d = n$), each component of the criminal network is complete in a pairwise stable Nash equilibrium.

Lemma

In any strongly stable Nash equilibrium, the equilibrium partition of agents "preserves" the order of detection probability

$$\left\{\{1, ..., n_1\}, \{n_1 + 1, ..., n_1 + n_2\}, ...\{\sum_{i=1}^{k-1} n_i + 1, ..., \sum_{i=1}^{k} n_i\}\right\}$$

where $n \equiv \sum_{i=1}^{k} n_i$ and agents are labeled such that $\beta_1 \leq ... \leq \beta_n$.

Equilibrium Characterization: Full Cascade

Proposition

There exists a generically unique strongly stable Nash equilibrium with the equilibrium partition $\{\{1,2,...,n_0\},\{n_0+1\},\{n_0+2\},...,\{n\}\}$ *and*

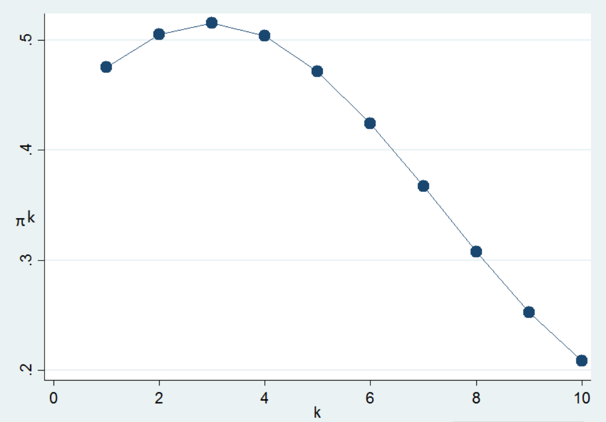
$$n_0 = \max\left\{\arg\max_{k\in N} \pi^k\right\},$$

where π^k *is the individual payoff of a complete component formed by the first* k *agents,*

$$\pi^k = \frac{1}{2}\left(\frac{1}{1-(k-1)\lambda}\right)^2 \Pi_{i=1}^k(1-\beta_i).$$

Equilibrium Characterization: Full Cascade

A numerical example: $n = 10; \beta_k = k/20; \lambda = 0.08$



Equilibrium Characterization: Partial Cascade

Proposition

*Those players who are isolated in the **strongly stable** Nash equilibrium under full cascade of detection remain isolated in **any pairwise stable** Nash equilibrium under partial cascade of detection.*

Detection Policy: Full Cascade

The government's decision problem

$$\min_{\beta\in\mathbb{R}^n_+:\ \sum_{i=1}^n \beta_i\le B} \sum_{i=1}^n \left(x_i(\beta,\lambda) - \frac{1}{2}x_i^2(\beta,\lambda) + \lambda \sum_{j=1}^n g_{ij}(\beta,\lambda)x_i(\beta,\lambda)x_j(\beta,\lambda) \right)$$

Equivalently,

$$\min_{\beta\in\mathbb{R}^n_+:\ \sum_{i=1}^n \beta_i\le B} n_0(\beta)$$

Detection Policy: Full Cascade

Proposition

Under full cascade of detection, the government can keep each agent isolated in the strongly stable Nash equilibrium if and only if

$$B > B_1 \equiv n - 1 - \sum_{k=2}^{n} \left(\frac{1-(k-1)\lambda}{1-(k-2)\lambda} \right)^2,$$

and the optimal allocation of the detection budget is given by $\beta_1 = 0$ *and*

$$\beta_k = 1 - \left(\frac{1-(k-1)\lambda}{1-(k-2)\lambda} \right)^2 + \frac{B - B_1}{n-1}, \quad k = 2, 3, ..., n.$$

Detection Policy: Full Cascade

Corollary

Under full cascade of detection, the government can keep the size of the largest component of the criminal network in the strongly stable Nash equilibrium to be $S \in \{2, 3, ..., n-1\}$ *if and only if*

$$B > B_S \equiv n - S - \sum_{k=S+1}^{n} \left(\frac{1-(k-1)\lambda}{1-(k-2)\lambda} \right)^2 ,$$

and the optimal allocation of the detection budget is given by $\beta_k = 0$ *for* $k \leq S$ *and*

$$\beta_k = 1 - \left(\frac{1-(k-1)\lambda}{1-(k-2)\lambda} \right)^2 + \frac{B - B_S}{n - S}, \quad k = S+1, S+2, ..., n.$$

Relation to the Literature

- Crime organizations and detection policy
  - Ballester, Calvo-Armengol, and Zenou (2006): “key-player” policy
  - Garoupa (2007): severe law enforcement could backfire
  - Baccara and Bar-Isaac (2008): optimal information structure
- Network games with local complementarities
  - Baetz (2014): one-sided link formation
  - Hiller (2014): two-sided link formation
  - Belhaj, Bervoets, and Deroian (2014): efficient network structures

Extension: Degree of Cascade

Proposition

Those players who are isolated in the strongly stable Nash equilibrium under full cascade of detection ($d = n$) *remain isolated in any strongly stable Nash equilibrium under positive degree of cascade* ($d \in \{1, 2, ..., n\}$).

Extension: Outside Option

Proposition

Let $B_n = 0$. If the detection budget $B \in [B_{\ell+1}, B_\ell)$[a] for $\ell \in \{1, 2, ..., n-1\}$, the government can incentivize all agents to opt out if and only if $\frac{1-(B-B_{\ell+1})}{2(1-\ell\lambda)^2} < \pi_0$ with the allocation of the detection budget given by $\beta_k = 0$ for $k \le \ell$, $\beta_{\ell+1} = B - B_{\ell+1}$, and $\beta_k = 1 - \left(\frac{1-(k-1)\lambda}{1-(k-2)\lambda}\right)^2$ for $k > \ell + 1$.

[a] Recall that $B_S \equiv n - S - \sum_{k=S+1}^{n} \left(\frac{1-(k-1)\lambda}{1-(k-2)\lambda}\right)^2$ for $S \in \{1, 2, ..., n-1\}$.

Conclusion

- Higher degree of cascade of detection may backfire. Higher degree of cascade of detection could yield a denser criminal network.
- Optimal budget allocation is highly asymmetric among ex ante identical agents.
- The results continue to hold under general degree of cascade of detection and introduction of outside option.

Thank you!